Exam Program Correctness, April, 7th 2015, 9:00-12:00h.

- This exam consists of three problems. Problem 1 is worth 20 points, problem 2 is worth 30 points, and problem 3 is worth 40 points. You get 10 points for not misspelling your name and student number.
- Give complete annotations, and linear proofs. Use a pen. Do not use a pencil!
- The exam is a closed book exam. You are not allowed to use the reader, slides, notes, or any other material.
- Do not hand in scratch paper!

Problem 1 (20 pt).
Design an annotated command S that satisfies the Hoare triple:

$$\{\ P:\ X \geq 0\ \wedge\ (p-2=X\ \vee\ p=-X)\ \wedge\ p^2+q=Y\ \}\ \ S\ \ \{\ Q:\ p=X\ \wedge\ p^2+q=Y\ \}$$

Problem 2 (30 pt). Design and prove the correctness of a command T that satisfies

const $n:\mathbb{N}$, $\quad a:$ **array** $[0..n)$ **of** $\mathbb{Z}$;
var $x:\mathbb{Z}$;
$\quad\{\ P:\textbf{true}\ \}$
T
$\quad\{\ Q: x=\Pi\{\Sigma\{a[j]\cdot a[k]\mid j,k:0\leq j\leq k<i\}\mid i:0\leq i<n\}\ \}$.

The time complexity of the command S must be linear in n. Start by defining (a) suitable helper function(s) and the corresponding recurrence(s).

Problem 3 (40 pt). Given is a two-dimensional array a that is *decreasing* in its first argument and *ascending* in its second argument. Consider the following specification:

const $n,w:\mathbb{N}$, $a:$ **array** $[0..n)$ **of** $\mathbb{N}$;
var $z:\mathbb{N}$;
$\quad\{P:\ Z=\#\{(i,j)\mid i,j:0\leq i\ \wedge\ 0\leq j\ \wedge\ i+j<n\ \wedge\ a[i,j]=w\}\ \}$
U
$\quad\{Q:\ Z=z\}$

(a) Make a sketch in which you clearly indicate where the array is high, low, and how a contour line goes.

(b) Define a function $F(x,y)$ that can be used to compute Z. Determine the relevant recurrences for $F(x,y)$, including the base cases.

(c) Design a command U that has a linear time complexity in n. Prove the correctness of your solution. [Note: you can only receive points for part (c) if the recurrences in part (b) are correct]